Systematic Risk in Supply Chain Networks

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1. Introduction

Identifying and managing the drivers of demand uncertainty is an important topic in supply chain management. Researchers have modeled demand as a function of its causal, time-series, and idiosyncratic components (e.g., Aviv 2003). They have evaluated the propagation of these components upstream in the supply chain through ordering decisions, assessed the value of information sharing, and investigated phenomena such as production smoothing and the bullwhip effect (Lee et al. 2000, Bray and Mendelson 2012). One important but understudied component of demand uncertainty is the systematic market-driven risk in demand. This factor represents the demand uncertainty associated with the overall economy. Its knowledge is useful for financial and operational hedging (Gaur and Seshadri 2005, Chod et al. 2010), and for valuation because financial theory prices risky assets according to their correlation with the broad market index. A key characteristic of systematic risk is that it is common across firms. Unlike idiosyncratic risk, it cannot be diversified away by pooling demand streams across firms. Therefore, systematic risk may not be exogenous to the supply chains of firms. Its propagation upstream in the supply chain should depend on the ordering decision rules used by firms as well as the network structure of the supply chain. In other words, the composition of demand volatility, one part due to systematic risk and the other due to reasons uncorrelated with the market, should change as we move upstream in a supply chain.

In this paper, we investigate the systematic risk in demand for different industries and firms in the U.S. economy, including retail trade, wholesale trade, and manufacturing sectors, and analyze how the structure of the supply network mediates the composition of demand uncertainty into systematic and idiosyncratic risk components. We consider sales as a proxy for demand throughout the paper guided by data considerations. We define the systematic risk in sales as the correlation coefficient of sales change with the contemporaneous return on a broad financial market index, i.e., the market return. Sales change is defined as the change in deseasonalized and price-adjusted sales over a given time window. Market return is computed as the return on the U.S. value-weighted market index.
Consider the following examples of supply linkages constructed from our data set. The estimates of systematic risk in sales for retailers Costco Wholesale Corp., Target Corp., and Wal-Mart Stores Inc. are 0.482, 0.00182, and 0.00190, respectively. Their supplier Procter & Gamble Co. has a systematic risk of 0.64, and its supplier 3M Co. has a systematic risk of 0.89. In another example, Hewlett-Packard Co. has a systematic risk in sales of 0.14; its suppliers Advanced Micro Devices and Intel Corp. have 0.36 and 0.52, respectively; and its supplier Applied Materials Inc. has 0.63. These examples illustrate the existence of a systematic market-associated component in the sales of these firms. They also motivate our research question: How does systematic risk propagate in supply chains through ordering decisions and supply linkages? More broadly, Figure 1 shows the evolution of sales and VWMI for the U.S. retail, wholesale trade, and manufacturing industries using monthly data for the years 1992–2008 and 12-month time windows. We compute the systematic risk for these industries and find that it increases from retailers to wholesalers to manufacturers, with values of 0.331, 0.415, and 0.502, respectively. The result also holds at the industry segment level defined by three- or four-digit North American Industry Classification System (NAICS) codes, as well as at the firm level.

For retailers, sales change is expected to be correlated with market return because end-consumer demand depends on the wealth of consumers, which in turn varies with market return. This argument follows from the permanent income hypothesis of Friedman (1957) and has been tested in several papers in macroeconomics, such as Ando and Modigliani (1963) and Ludvigson et al. (1998), and more recently in operations management by Osadchiy et al. (2013). One might expect the same logic to hold for wholesalers and manufacturers as well because they supply products to retailers. However, their correlation coefficients, i.e., systematic risk, will be modulated by supply chain dynamics. It is not obvious a priori whether the correlation should increase or decrease depending on the supply chain structure of a firm or industry. We investigate three potential mechanisms that might impact the correlation with different implications for supply chain managers.

First, can demand signal processing done by managers in generating their production orders lead to an amplification of systematic risk in supply chains? If managers distinguish between different sources of uncertainty in demand, then they can emphasize systematic risk more than idiosyncratic noise. However, they may also influence the total variance of order streams in accordance with either production smoothing or the bullwhip effect. Production smoothing results in decreased variance of orders placed upstream, which, depending on the changes in covariance, can lead to an increase or a decrease in the correlation coefficient. The bullwhip can lead to a decrease in the correlation if the covariance between production orders and market return grows slower than the variance of orders. We find that for the majority of the industries, production time series are less correlated with market returns. In other words, industries on average decrease the systematic risk in their order processes. We find a similar effect of nonamplification of the systematic risk at the firm level. Thus, demand signal processing does not explain our observation of amplification of systematic risk upstream in the supply chain.

The second mechanism is based on the recognition that supply chains are in fact supply networks.
Each agent in a supply chain receives orders from many customers. The total shipments from an industry represent a response to the aggregate stream of orders. Is there any reason to believe that the correlation of the aggregate order stream with the market is higher than the correlation of the each of the orders that it consists of? The answer is yes, and the reason is purely statistical: if one sums up random variables each having some positive correlation with the market and some independent noise, the noise will attenuate with respect to the signal, but the correlated components will add up. Moreover, the degree of the effect will depend on the number and sizes of customers. We test this hypothesis. Using the IO tables for the U.S. economy, we reconstruct the aggregated order process for each industry based on its bill of materials and the production processes of its downstream industries. These reconstructed order processes demonstrate an increase in the correlation coefficient even though the downstream production processes had low correlation coefficients to begin with. The aggregation hypothesis implies that a more dispersed customer base is associated with higher systematic risk. Using the year-to-year variation in IO tables, we test and find support for this hypothesis on industry-level data. At the firm level, we also find support for this hypothesis using newly available information on customers of major companies. Collectively, this evidence supports aggregation of orders over customers as a driver of amplification of systematic risk in supply chains.

The third mechanism relates to the aggregation of order flows over time. A market shock in one period may affect sales over several periods due to lead times and time lags in managerial decision making. When aggregated over time, these systematic effects amplify, whereas the idiosyncratic noise in sales cancels, thus leading to an increase in systematic risk. We find empirical support for this mechanism at the industry and firm levels, with manufacturers having the strongest effect of aggregation of orders over time.

The main contribution of our paper is to describe the phenomenon of systematic risk in sales and investigate mechanisms leading to its amplification in supply networks. Studying risk at the network level, we believe, is an important methodological contribution that uncovers new insights not available at the buyer–supplier dyad level. In particular, we show that the systematic and idiosyncratic components of sales volatility depend on the supply chain structure in a firm or an industry. We explore several managerial implications of systematic risk. First, systematic risk is priced by financial market through the cost of capital. On average, an increase in systematic risk by 0.1 increases equity premium of the respective firm by 0.25%. The estimates from our model can be used to assess the effect of the structure of the supply chain of a firm on its systematic risk in sales, and hence its equity premium. Second, our estimates of the effect of time aggregation on systematic risk imply that supply chain initiatives that involve changes in the length of planning horizon or lead time (e.g., outsourcing or quick response) change the systematic risk and therefore must be valued with a correspondingly adjusted discount rate. Third, since customer-base dispersion is an important driver of the systematic risk, a decision to source from a supplier can change the systematic risk of a supplier. For instance, whether a firm chooses a dedicated or a shared supplier will affect the supplier’s systematic risk, cost of capital, and exposure to economic conditions. These factors will in turn affect the buyer by altering the risk and cost structure of the supplier; e.g., the cost of capital of the supplier will affect its inventory holding cost and other operational costs, which will affect the buyer’s cost.

2. Literature Review

Our paper is related to the literature on managing demand risk with financial instruments, production smoothing, the bullwhip effect, and supply chain networks. Our definition of systematic risk is identical to that in Gaur and Seshadri (2005) and Chod et al. (2010). Both papers assume demand correlated with the contemporaneous financial market return. Gaur and Seshadri (2005) derive an optimal hedging policy that minimizes the variance of profit. Analyzing the value of the firm under operational and financial hedging, Chod et al. (2010) derive conditions under which the two are complements or substitutes. In addition to hedging with financial instruments, many specific methods for reducing and managing demand risk have been developed in the literature under the umbrella of demand pooling, operational flexibility, and operational hedging. They include flexible capacity (Jordan and Graves 1995, Van Mieghem 1998, Goyal and Netessine 2007), delayed differentiation (Lee et al. 1993), and geographical pooling (Eppen and Schrage 1981).

The focus of production smoothing and the bullwhip effect studies is on total demand volatility. The premise of production smoothing is that businesses use inventory buffers to smooth idiosyncratic demand shocks, yet the phenomenon has been difficult to confirm empirically. Several explanations have been offered, including measurement errors and aggregation biases, and notably, the bullwhip effect. Theoretical research on the bullwhip effect predicts an increase in the variance of demand upstream in the supply chain. Investigating this effect empirically in industry-level data, Cachon et al. (2007) discover that most
industries smooth seasonality, but there is evidence for the bullwhip effect in deseasonalized data. Bray and Mendelson (2012) study the bullwhip effect in firm-level data and find that there is evidence for the existence of the bullwhip effect for wholesale trade, manufacturing, and resource extraction sectors of the economy, but not for retail trade. Recent attempts to reconcile production smoothing theory with the bullwhip effect include the work of Bray and Mendelson (2015), who observe that businesses do smooth production if one controls for the predicted presence of the bullwhip effect. Motivated by the mixed evidence supporting the bullwhip effect in aggregated data, Chen and Lee (2012) analyze the effect of time aggregation and cross-sectional aggregation on the ability to measure the bullwhip effect.

Our paper focuses on the systematic, market-associated component of demand uncertainty. It uses a similar data set and methodology as Cachon et al. (2007), but focuses on the correlation coefficients of changes in sales and production with market return, not their variances. Its main contribution is in analyzing the impact of supply network structure on systematic risk. Cachon et al. (2007, p. 466) identify the need to construct supply networks to evaluate amplification ratios at a finer level of granularity. We conduct such analysis by using input–output matrices and investigating the effect of aggregation on systematic risk. Thus, our paper utilizes the framework for input–output analysis developed by Leontief (1936, 1986), which has been instrumental for studies in many disciplines, including macroeconomics, finance, and operations management.

The recent literature shows a growing focus on the econometric analysis of supply chain networks and their implications. Acemoglu et al. (2012) show that idiosyncratic demand shocks can amplify due to the network structure of economy. Cohen and Frazzini (2008) and Menzly and Ozbas (2010) use supply linkages between firms to study the predictability of their stock returns. They show that financial news travels slowly along supply chains, so that lagged customers’ stock returns are predictive of suppliers’ stock returns. In operations management, Barker and Santos (2010) use input–output data to model production breakdowns and investigate the effect of inventory on the speed of recovery. Wang et al. (2014) study how supply chain disruptions propagate through a supply network of high-technology companies, Bellamy et al. (2014) show that supply networks influence firms’ innovations, and Wu and Birge (2014) study how supply chain structure is associated with firms’ stock return.

Supply chain-derived measures have also been used in recent empirical research to explain firms’ operational and financial performance. Cachon and Olivares (2010) identify the number of dealerships in an automaker’s distribution network as an important driver of finished goods inventory. Jain et al. (2014) find that a more dispersed supplier base is associated with decreased inventory investment in global supply chains. Another related paper, Patatoukas (2011), studies the effect of customer concentration on suppliers’ stock returns and finds that suppliers with a concentrated customer base typically have higher stock returns and lower operational expenses. In addition, it finds that companies with low customer concentration have a larger share of delistings due to bankruptcy or insolvency. Collectively, these findings show that supply chain structure is important for the financial performance and risk of a firm.

The finance and accounting literature links systematic risk to the cost of capital of a firm. Systematic risk and its determinants have long been studied in this literature. For stock returns, the systematic, market-associated risk is usually represented by “beta.” Beaver et al. (1970) study how firms’ betas are associated with accounting parameters, such as dividends payout, asset size, earnings variability, and earnings covariability. Citing the need for theoretical models relating betas to the accounting numbers, Bowman (1979) derives the relationship between betas, leverage, and the accounting (earnings-based) beta. Although many potential determinants of systematic risk have been studied in the literature, to the best of our knowledge, this paper is the first to relate systematic risk to the placement and structure of a company’s supply chain. Given the recent advances in supply chain management, it is important to understand their impact on the risk profile of the firm. We quantify the impact of changes in customer-base concentration and planning horizon on the systematic risk and risk-adjusted valuation of the firm.

3. Data

We use data for the U.S. economy obtained from the following sources:

(i) monthly surveys of manufacturing, wholesale trade, and retail trade sectors conducted by the U.S. Census Bureau (U.S. Census Bureau 2010c, d, e);
(ii) estimated annual gross margins reported by the U.S. Census Bureau for manufacturing, wholesale trade, and retail trade sectors (U.S. Census Bureau 1997, 2002, 2008a, b);
(iii) price deflator data (Bureau of Economic Analysis 2010b);
(iv) input–output tables from the Bureau of Labor Statistics (BLS) for 195 industries for years 1993–2010 (Bureau of Labor Statistics 2013);
(v) firm-level data on major customers for manufacturing, retail, and wholesale trade companies, constituents of the S&P 500 index (Bloomberg 2013);
(vi) firm-level quarterly sales, inventory, and cost of goods sold data for manufacturing, retail, and wholesale trade companies, constituents of the S&P 500 index (Standard & Poor’s 2014);
(vii) historical daily returns and levels of the value-weighted market index including distributions (Center for Research in Security Prices 2014a);
(viii) historical daily returns and closing market capitalizations of U.S. publicly listed firms in the manufacturing, wholesale trade, and retail trade sectors (Center for Research in Security Prices 2014b).

We use the first three data sources to construct sales, inventory, and production series for each industry; the fourth to map supply chain flows across industries; the next two data sets to verify the results at the firm level; and the last two data sets to compute systematic risk in sales and estimate the values of beta for all industries. Usually, the wholesale trade sector is considered to be upstream of retail trade, and manufacturing upstream of wholesale trade. However, flows may not always follow this direction. Thus, we use the data on input–output matrices and customer–supplier pairs ((iv)–(v) above) to formally map supply chain flows across industries and firms, respectively. We describe the construction of key variables of interest below.

3.1. Sales, Inventory, and Production

We follow the method described by Cachon et al. (2007) to construct these time series. The main steps in this method deal with the classification of the data into industry segments and the adjustment of sales data for gross margin and price deflation. We obtain data on seasonally adjusted sales and inventories from the monthly surveys of manufacturing, wholesale trade, and retail trade sectors conducted by the U.S. Census Bureau (U.S. Census Bureau 2010c, d, e). Table 1 lists the industry segments in our study, see the second column and the table notes. For manufacturers, the report gives data on volume of shipments. We treat shipments as equivalent to sales. We use data from January 1992 to December 2007, yielding 192 monthly data points in each time series. The U.S. Census Bureau classifies the data for each sector into industry segments, using the M3 classification for manufacturing and the NAICS classification for retail and wholesale trade. The mapping from M3 to NAICS segments is one to one in some instances, one to many in others, and many to one in the rest; that is, some of the M3 segments directly correspond to NAICS segments, some represent a combination of disjoint NAICS segments, and still others represent a decomposition of NAICS segments. This mapping is based on U.S. Census Bureau (2010a) and provided in the third and fourth columns of Table 2, with the composition of aggregated NAICS segments further explained in the notes to the table. This mapping is also used to apply price deflators and aggregate IO tables.

Because inventories and sales are valued at different prices for each industry segment, we adjust sales data using estimated annual gross margins to control for this difference. Gross margins are available for retail and wholesale trade for the years 1993–2007 (U.S. Census Bureau 2008a, b). For manufacturers, we compute gross margins from the sales and cost of goods sold data available from the 1997 and 2002 economic census reports (U.S. Census Bureau 1997, 2002). When price margin data for a year are missing, we use data from the closest year. A comparison of values computed from year 1997 and 2002 economic census reports shows that the gross margins for the manufacturing industry segments are fairly stable over time.

We collect annual price deflator data (Bureau of Economic Analysis 2010b) and adjust each time series for inflation and/or price fluctuations to conduct comparisons across years. We use price deflators rather than price inflation indices because price deflators vary with changes in the product mix produced by an industry, whereas price inflation indices do not. Price deflators are provided according to the NAICS classification. There is an exact match between price deflator segments and segments of retailers and wholesalers. To apply the price deflators to the M3 manufacturing series, we use the mapping provided by U.S. Census Bureau (2010a). Our approach to account for gross margins and inflation is the same as that used by Cachon et al. (2007).

3.2. Input–Output Matrices

We obtain the make and use IO tables from the BLS input–output accounts for years 1993–2010 (Bureau of Labor Statistics 2013). The make table lists the amounts in dollars of different commodities produced by each industry, with the diagonal representing the amount of primary commodity output for each industry. There is a one-to-one correspondence between an industry and a primary commodity output. The use matrix provides the amount of each commodity used as input by various industries and in final demand (consumption and net export). Final demand is represented as an industry with zero output and a nonzero column in the use matrix.

Industries in IO matrices are defined using the NAICS classification. Most industries correspond to

1 NAICS is a tree-like classification system. Disjoint means that the segments belong to nonoverlapping subtrees.

2 The Bureau of Economic Analysis provides monthly price deflator data in National Input and Product Accounts (NIPA) Tables 2AUI and 2BU1. Table 2AUI covers years 1967–1996, and Table 2BU1 covers years from 1997 onward.
Table 1 Correlation Measures for Industry Segments, Years 1992–2007

<table>
<thead>
<tr>
<th>NAICS/M3 code</th>
<th>( \rho_S )</th>
<th>( \rho_P )</th>
<th>( \text{Cov}_S )</th>
<th>( \rho_S )</th>
<th>( \text{Cov}_P )</th>
<th>( \rho_P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail trade industries(^a)</td>
<td>44, 45</td>
<td>0.331</td>
<td>0.172</td>
<td>174,285</td>
<td>0.022</td>
<td>116,385</td>
</tr>
<tr>
<td>Retail trade industries (excl. motor vehicles)(^a)</td>
<td>44, 45 excl. 441</td>
<td>0.196</td>
<td>0.103</td>
<td>72,160</td>
<td>0.142</td>
<td>48,836</td>
</tr>
<tr>
<td>Motor vehicle and parts dealers</td>
<td>441</td>
<td>0.252</td>
<td>0.171</td>
<td>105,240</td>
<td>0.035</td>
<td>75,714</td>
</tr>
<tr>
<td>Furniture, furnishings, electronics, and appliance stores</td>
<td>442, 443</td>
<td>0.152</td>
<td>0.145</td>
<td>11,930</td>
<td>0.252</td>
<td>14,927</td>
</tr>
<tr>
<td>Building material and garden equipment and supplies dealers</td>
<td>444</td>
<td>0.108</td>
<td>0.078</td>
<td>8,814</td>
<td>0.114</td>
<td>6,666</td>
</tr>
<tr>
<td>Food and beverage stores</td>
<td>445</td>
<td>0.159</td>
<td>0.117</td>
<td>8,833</td>
<td>0.309</td>
<td>7,654</td>
</tr>
<tr>
<td>Clothing and clothing accessories stores</td>
<td>448</td>
<td>0.291</td>
<td>0.044</td>
<td>10,514</td>
<td>0.011</td>
<td>3,336</td>
</tr>
<tr>
<td>General merchandise stores</td>
<td>452</td>
<td>-0.166</td>
<td>-0.227</td>
<td>-14,142</td>
<td>0.324</td>
<td>-29,649</td>
</tr>
<tr>
<td>Department stores(^a)</td>
<td>4521</td>
<td>0.168</td>
<td>0.014</td>
<td>10,179</td>
<td>0.277</td>
<td>1,340</td>
</tr>
</tbody>
</table>

**Merchant wholesale industries\(^a\)**

<table>
<thead>
<tr>
<th>NAICS/M3 code</th>
<th>( \rho_S )</th>
<th>( \rho_P )</th>
<th>( \text{Cov}_S )</th>
<th>( \rho_S )</th>
<th>( \text{Cov}_P )</th>
<th>( \rho_P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durable goods merchant wholesale industries(^a)</td>
<td>423</td>
<td>0.557</td>
<td>0.441</td>
<td>308,751</td>
<td>0.000</td>
<td>309,317</td>
</tr>
<tr>
<td>Motor vehicles, parts, and supplies wholesalers</td>
<td>4231</td>
<td>-0.138</td>
<td>-0.156</td>
<td>-19,537</td>
<td>0.175</td>
<td>-30,648</td>
</tr>
<tr>
<td>Furniture and home furnishings wholesalers</td>
<td>4232</td>
<td>0.323</td>
<td>0.197</td>
<td>6,717</td>
<td>0.000</td>
<td>5,743</td>
</tr>
<tr>
<td>Lumber and other construction materials wholesalers</td>
<td>4233</td>
<td>0.079</td>
<td>0.038</td>
<td>5,735</td>
<td>0.438</td>
<td>3,271</td>
</tr>
<tr>
<td>Professional and commercial equipment wholesalers</td>
<td>4234</td>
<td>0.290</td>
<td>0.270</td>
<td>82,985</td>
<td>0.023</td>
<td>86,748</td>
</tr>
<tr>
<td>Computers and software wholesalers(^a)</td>
<td>42343</td>
<td>0.212</td>
<td>0.207</td>
<td>142,874</td>
<td>0.111</td>
<td>151,380</td>
</tr>
<tr>
<td>Metal and mineral (except petroleum) wholesalers</td>
<td>423S</td>
<td>0.382</td>
<td>0.309</td>
<td>21,281</td>
<td>0.008</td>
<td>22,105</td>
</tr>
<tr>
<td>Electrical goods wholesalers</td>
<td>4236</td>
<td>0.470</td>
<td>0.428</td>
<td>83,499</td>
<td>0.008</td>
<td>93,810</td>
</tr>
<tr>
<td>Hardware and plumbing and heating equipment wholesalers</td>
<td>4237</td>
<td>0.373</td>
<td>0.247</td>
<td>12,133</td>
<td>0.011</td>
<td>10,935</td>
</tr>
<tr>
<td>Machinery, equipment, and supplies wholesalers</td>
<td>4238</td>
<td>0.615</td>
<td>0.467</td>
<td>70,136</td>
<td>0.000</td>
<td>68,858</td>
</tr>
<tr>
<td>Miscellaneous durable goods wholesalers</td>
<td>4239</td>
<td>0.278</td>
<td>0.303</td>
<td>25,541</td>
<td>0.005</td>
<td>29,846</td>
</tr>
<tr>
<td>Nonreturnable goods merchant wholesale industries(^a)</td>
<td>424</td>
<td>-0.421</td>
<td>-0.086</td>
<td>-31,121</td>
<td>0.367</td>
<td>-32,707</td>
</tr>
<tr>
<td>Paper and paper products wholesalers</td>
<td>4241</td>
<td>-0.012</td>
<td>0.035</td>
<td>-0.423</td>
<td>0.914</td>
<td>1,348</td>
</tr>
<tr>
<td>Drugs and drugstores’ sundries wholesalers</td>
<td>4242</td>
<td>-0.453</td>
<td>-0.402</td>
<td>-57,084</td>
<td>0.002</td>
<td>-63,000</td>
</tr>
<tr>
<td>Apparel, piece goods, and notions wholesalers</td>
<td>4243</td>
<td>0.021</td>
<td>-0.041</td>
<td>0.925</td>
<td>0.855</td>
<td>-2,897</td>
</tr>
<tr>
<td>Grocery and related products wholesalers</td>
<td>4244</td>
<td>0.104</td>
<td>0.118</td>
<td>12,706</td>
<td>0.582</td>
<td>16,306</td>
</tr>
<tr>
<td>Farm product raw materials wholesalers</td>
<td>4245</td>
<td>-0.003</td>
<td>0.027</td>
<td>-0.479</td>
<td>0.976</td>
<td>6,546</td>
</tr>
<tr>
<td>Chemical and allied products wholesalers</td>
<td>4246</td>
<td>0.066</td>
<td>0.028</td>
<td>1,803</td>
<td>0.457</td>
<td>1,071</td>
</tr>
<tr>
<td>Petroleum and petroleum products wholesalers</td>
<td>4247</td>
<td>-0.080</td>
<td>-0.078</td>
<td>-14,611</td>
<td>0.597</td>
<td>-15,654</td>
</tr>
<tr>
<td>Beer, wine, and distilled alcoholic beverages wholesalers</td>
<td>4248</td>
<td>0.061</td>
<td>0.043</td>
<td>1,458</td>
<td>0.600</td>
<td>1,653</td>
</tr>
<tr>
<td>Miscellaneous nondurable goods wholesalers</td>
<td>4249</td>
<td>0.316</td>
<td>0.277</td>
<td>26,452</td>
<td>0.010</td>
<td>29,298</td>
</tr>
</tbody>
</table>

**Total manufacturing\(^a\)**

| MTM | 0.502 | 0.443 | 727,646 | 0.012 | 662,517 | 0.049 |

Notes: For manufacturing, the nonoverlapping segments are reported at the industry-segment level. M3 codes for such segments end with “S.” The M3 survey contains more detailed data for industry subsegments, e.g., for the segment 12S, separate series for subsegments 12A (Beverage Manufacturing) and 12B (Tobacco Manufacturing) are available. These data are used the input–output analysis (Table 2).

\(^a\)Aggregate segments and subsegments are excluded from the analysis of \( \rho_S \) and \( \rho_P \) to avoid double counting.
Table 2  
Correlation of Sales Change and Market Returns Computed via Census Data ($\rho_S$), IO Analysis ($\rho_O$), Sales Volatility ($\sigma_S$), and Beta of Industry Portfolios

<table>
<thead>
<tr>
<th>Industry (IO, aggregated)</th>
<th>NAICS (IO)</th>
<th>M3</th>
<th>$\rho_S$</th>
<th>$\rho_O$</th>
<th>$\sigma_S$</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food manufacturing</td>
<td>1</td>
<td>311</td>
<td>0.264</td>
<td>0.297</td>
<td>0.034</td>
<td>0.522</td>
</tr>
<tr>
<td>Beverage manufacturing</td>
<td>2</td>
<td>3121</td>
<td>0.038</td>
<td>0.309</td>
<td>0.101</td>
<td>0.462</td>
</tr>
<tr>
<td>Tobacco manufacturing</td>
<td>3</td>
<td>3122</td>
<td>0.546</td>
<td>0.058</td>
<td>0.185</td>
<td>0.504</td>
</tr>
<tr>
<td>Textile mills</td>
<td>4</td>
<td>313</td>
<td>0.027</td>
<td>0.122</td>
<td>0.065</td>
<td>0.652</td>
</tr>
<tr>
<td>Textile product mills</td>
<td>5</td>
<td>314</td>
<td>0.349</td>
<td>0.122</td>
<td>0.067</td>
<td>0.826</td>
</tr>
<tr>
<td>Apparel manufacturing</td>
<td>6</td>
<td>315</td>
<td>0.465</td>
<td>0.413</td>
<td>0.080</td>
<td>0.919</td>
</tr>
<tr>
<td>Leather and allied product manufacturing</td>
<td>7</td>
<td>316</td>
<td>0.431</td>
<td>0.387</td>
<td>0.111</td>
<td>0.777</td>
</tr>
<tr>
<td>Wood product manufacturing</td>
<td>8</td>
<td>321</td>
<td>0.217</td>
<td>0.265</td>
<td>0.063</td>
<td>0.850</td>
</tr>
<tr>
<td>Pulp, paper, and paperboard mills</td>
<td>9</td>
<td>3221</td>
<td>0.140</td>
<td>0.168</td>
<td>0.066</td>
<td>0.663</td>
</tr>
<tr>
<td>Converted paper product manufacturing</td>
<td>10</td>
<td>3222</td>
<td>0.157</td>
<td>0.276</td>
<td>0.041</td>
<td>0.687</td>
</tr>
<tr>
<td>Printing and related support activities</td>
<td>11</td>
<td>323</td>
<td>0.476</td>
<td>0.691</td>
<td>0.036</td>
<td>0.440</td>
</tr>
<tr>
<td>Petroleum and coal products manufacturing</td>
<td>12</td>
<td>324</td>
<td>0.250</td>
<td>0.126</td>
<td>0.080</td>
<td>0.615</td>
</tr>
<tr>
<td>Basic chemical agg.</td>
<td>13</td>
<td>325X</td>
<td>0.072</td>
<td>0.232</td>
<td>0.044</td>
<td>0.648</td>
</tr>
<tr>
<td>Agricultural chemical manufacturing</td>
<td>14</td>
<td>3253</td>
<td>0.012</td>
<td>0.423</td>
<td>0.109</td>
<td>0.634</td>
</tr>
<tr>
<td>Pharmaceutical and medicine manufacturing</td>
<td>15</td>
<td>3254</td>
<td>0.058</td>
<td>0.193</td>
<td>0.074</td>
<td>0.821</td>
</tr>
<tr>
<td>Paint, coating, and adhesive manufacturing</td>
<td>16</td>
<td>3255</td>
<td>0.197</td>
<td>0.064</td>
<td>0.055</td>
<td>0.818</td>
</tr>
<tr>
<td>Plastics and rubber products manufacturing</td>
<td>17</td>
<td>326</td>
<td>0.364</td>
<td>0.363</td>
<td>0.045</td>
<td>0.828</td>
</tr>
<tr>
<td>Nonmetallic mineral product manufacturing</td>
<td>18</td>
<td>327</td>
<td>0.348</td>
<td>0.286</td>
<td>0.049</td>
<td>0.778</td>
</tr>
<tr>
<td>Iron agg.</td>
<td>19</td>
<td>331X</td>
<td>0.184</td>
<td>0.237</td>
<td>0.125</td>
<td>0.941</td>
</tr>
<tr>
<td>Foundries</td>
<td>20</td>
<td>3315</td>
<td>0.305</td>
<td>0.241</td>
<td>0.105</td>
<td>0.654</td>
</tr>
<tr>
<td>Fabricated metal agg.</td>
<td>21</td>
<td>332</td>
<td>0.457</td>
<td>0.509</td>
<td>0.063</td>
<td>0.686</td>
</tr>
<tr>
<td>Agriculture, construction, and mining machinery manufacturing</td>
<td>22</td>
<td>3331</td>
<td>0.214</td>
<td>0.632</td>
<td>0.158</td>
<td>0.878</td>
</tr>
<tr>
<td>Industrial machinery agg.</td>
<td>23</td>
<td>333X</td>
<td>0.281</td>
<td>0.576</td>
<td>0.234</td>
<td>1.444</td>
</tr>
<tr>
<td>HVAC and commercial refrigeration equipment manufacturing</td>
<td>24</td>
<td>3334</td>
<td>0.279</td>
<td>0.504</td>
<td>0.122</td>
<td>0.659</td>
</tr>
<tr>
<td>Metalworking machinery manufacturing</td>
<td>25</td>
<td>3335</td>
<td>0.227</td>
<td>0.540</td>
<td>0.133</td>
<td>0.939</td>
</tr>
<tr>
<td>Engines and other machinery agg.</td>
<td>26</td>
<td>333Y 33F, G, J, K–N</td>
<td>0.416</td>
<td>0.572</td>
<td>0.088</td>
<td>0.933</td>
</tr>
<tr>
<td>Computer and peripheral equipment manufacturing</td>
<td>27</td>
<td>3341</td>
<td>0.421</td>
<td>0.639</td>
<td>0.172</td>
<td>1.680</td>
</tr>
<tr>
<td>Audio, video, and communications equipment manufacturing</td>
<td>28</td>
<td>334X 34D, E, F</td>
<td>0.548</td>
<td>0.656</td>
<td>0.255</td>
<td>1.655</td>
</tr>
<tr>
<td>Semiconductor and other electronic component manufacturing</td>
<td>29</td>
<td>3344</td>
<td>0.519</td>
<td>0.698</td>
<td>0.230</td>
<td>1.771</td>
</tr>
<tr>
<td>Electronic instrument manufacturing</td>
<td>30</td>
<td>3345</td>
<td>0.272</td>
<td>0.620</td>
<td>0.093</td>
<td>0.904</td>
</tr>
<tr>
<td>Manufacturing and reproducing magnetic and optical media</td>
<td>31</td>
<td>3346</td>
<td>0.446</td>
<td>0.610</td>
<td>0.203</td>
<td>1.192</td>
</tr>
<tr>
<td>Electric lighting equipment manufacturing</td>
<td>32</td>
<td>3351</td>
<td>0.181</td>
<td>0.583</td>
<td>0.127</td>
<td>0.818</td>
</tr>
<tr>
<td>Household appliance manufacturing</td>
<td>33</td>
<td>3352</td>
<td>0.040</td>
<td>0.481</td>
<td>0.080</td>
<td>0.883</td>
</tr>
<tr>
<td>Electrical equipment manufacturing</td>
<td>34</td>
<td>3353</td>
<td>0.350</td>
<td>0.560</td>
<td>0.096</td>
<td>0.921</td>
</tr>
<tr>
<td>Other electrical equipment and component manufacturing</td>
<td>35</td>
<td>3359 34H, 35D</td>
<td>0.553</td>
<td>0.704</td>
<td>0.157</td>
<td>1.151</td>
</tr>
<tr>
<td>Motor vehicle manufacturing</td>
<td>36</td>
<td>3361 36A, B, C</td>
<td>0.014</td>
<td>0.179</td>
<td>0.116</td>
<td>1.124</td>
</tr>
<tr>
<td>Motor vehicle body, trailer, and parts manufacturing</td>
<td>37</td>
<td>336X 36D, E</td>
<td>0.150</td>
<td>0.071</td>
<td>0.068</td>
<td>0.810</td>
</tr>
<tr>
<td>Aerospace product and parts manufacturing</td>
<td>38</td>
<td>3364 36F–J</td>
<td>0.163</td>
<td>0.225</td>
<td>0.631</td>
<td>0.870</td>
</tr>
<tr>
<td>Other transportation equipment manufacturing</td>
<td>39</td>
<td>336Y 36K–N</td>
<td>0.167</td>
<td>0.009</td>
<td>0.120</td>
<td>0.691</td>
</tr>
<tr>
<td>Furniture and related product manufacturing</td>
<td>40</td>
<td>337</td>
<td>0.236</td>
<td>0.389</td>
<td>0.060</td>
<td>0.759</td>
</tr>
<tr>
<td>Misc agg.</td>
<td>41</td>
<td>3399</td>
<td>0.200</td>
<td>0.056</td>
<td>0.038</td>
<td>0.789</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>42</td>
<td>42</td>
<td>—</td>
<td>0.415</td>
<td>0.365</td>
<td>0.024</td>
</tr>
<tr>
<td>Retail trade</td>
<td>43</td>
<td>44–45</td>
<td>—</td>
<td>0.331</td>
<td>0.185</td>
<td>0.019</td>
</tr>
<tr>
<td>Manufacturing aggregatea</td>
<td>44</td>
<td>31–33</td>
<td>—</td>
<td>—</td>
<td>0.502</td>
<td>—</td>
</tr>
<tr>
<td>All others</td>
<td>45</td>
<td>—</td>
<td>—</td>
<td>0.420</td>
<td>0.521</td>
<td>—</td>
</tr>
</tbody>
</table>


aManufacturing aggregate estimates are presented for reference but are not included in the analysis of $\rho_S$ and $\rho_O$ and $\rho_S$ and beta to avoid double counting.

bDuring 1992–2007, tobacco manufacturing experienced a steady decline in production due to increasing regulations and public health campaigns (American Lung Association 2011). A large drop in production occurred in 2000–2002 and coincided with the economic downturn, resulting in an abnormally large $\rho_S$.
Notes. Arrow size is proportional to the shipment volume. In panel (a), the node radius is proportional to the total industry output. Shipments to automotive manufacturing from motor vehicle bodies and trailers, machinery, fabricated metal, and rubber and plastic manufacturers are highlighted. Panel (b) shows food manufacturing firms and their customers. Wal-Mart is highlighted.

Sources. (a) IO tables for the year 2002; (b) Bloomberg (2013).

(U.S. Census Bureau 2010a). During this aggregation, we add up rows of the matrices with four-digit NAICS codes corresponding to an M3 segment, and replace them with one aggregate row. The same is done for commodities. Since our primary focus is on the retail, wholesale, and manufacturing sectors, we collapse the parts of the matrix that are not related to retail, wholesale, and manufacturing sectors into one row (for industries) and one column (for commodities). The aggregated industries are agriculture, mining, construction, utilities, financial, professional services, information technology, other industries, and government services. The elements in the row are the amounts of commodities produced or used by all other industries; the column represents the amount of all other aggregate commodities produced or used by each industry. The aggregated make-and-use matrices contain 44 industries and 44 commodities, corresponding to 41 industries in manufacturing, plus wholesale trade, retail trade, and all other industries. We then compute the industry-by-industry requirements matrix, which for each industry shows the requirement of inputs from each industry per $1 output. Section 5.2 describes the calculation of the requirements matrix.

To account for the final demand in the economy, we augment the requirements matrix by one row and one column, representing the final demand “industry.” The total final demand volume is obtained from the gross domestic product (GDP) quarterly series (U.S. Census Bureau 2010b), and the structure of final demand is obtained from the IO use matrix. GDP series is adjusted for price using GDP deflators (Bureau of Economic Analysis 2010a). The sales of all other industries are obtained from the IO tables, at the annual level, and then converted to monthly numbers and adjusted for price using GDP deflators. Figure 2(a) displays the interindustry flows in the U.S. economy for year 2002.

3.3. Firm-Level Data
We collect firm-level data for all firms that were constituents of the S&P 500 index on January 1, 2012, and are in the manufacturing, retail, and wholesale trade segments. There are 239 such firms. Bloomberg (2013) identifies the customers for each such firm and reports sales volumes when available. The time range of the reported relationships is from 2010 to 2012. These data are not limited by the 10% sales revenue restriction imposed by the Compustat segment files. Thus, the average number of customers reported by firms in our data set is 48 and the maximum is 414 (for Hewlett-Packard), whereas the average for the

3 Annualized GDP volume is reported quarterly. We transform this into monthly series by dividing the GDP volume by 12 and assigning it to the three respective months. Monthly GDP deflators are then applied.
Compustat segment files is less than 2. Figure 2 shows the nature of our firm-level data set and the depth of supply chain links. Note that the number of customer and supplier links varies across firms, and many supply chains have three or more tiers. Thus, the data set enables us to relate systematic risk to the characteristics of the supply network of a firm.

We also collect quarterly sales, inventory, and cost of goods sold data for each firm in our data set from 2009 to 2013 (Standard & Poor’s 2014). Analogous to the industry-level data, we adjust sales for margins and price inflation. We use price deflators corresponding to the NAICS segment and time period for each firm-year observation.

3.4. Stock Prices and Market Return Data

We collect daily data on historical returns for the VWMI for the years 1992 to 2013. This index is a value-weighted composition of closing prices of all stocks traded on the New York Stock Exchange, American Stock Exchange, and NASDAQ, including dividends and distributions. We use it as the broadest financial indicator available; it is commonly used as a proxy for market portfolio in research in asset pricing. Monthly data are used to estimate the covariance of sales with market returns, and daily data are used to compute beta to study the relationship between systematic risk and beta. To compute beta, we further collect historical daily stock returns for public firms in retail trade, wholesale trade, and manufacturing for the years 1992–2007. Then we compute the returns on value-weighted portfolios of companies from each industry segment. Closing market capitalization on the previous trading day is used as the weight. Returns on these industry segment portfolios are used to compute the beta for each industry segment and test the relationship between betas and systematic risks. For the firm-level analysis, we use betas for years 2009–2013, obtained from Center for Research in Security Prices (2014a).

4. Amplification of Systematic Risk

In this section, we define and estimate the systematic risk in sales. Let \( S(i, t) \) denote the sales revenue for firm or industry \( i \) in time period \( t \), let \( SC(i, t, T) = S(i, t + T) - S(i, t) \) denote the sales change for firm or industry \( i \) over the time window \([t, t + T]\), let \( VWMI(t) \) denote the value of the market index at the end of time period \( t \), and let \( r(t, T) = \ln VWMI(t + T) - \ln VWMI(t) \) denote the realized return on the market index during \([t, t + T]\). The index \( i \) will refer to firm \( i \) when we apply our model to firm-level data or to industry \( i \) when we present industry-level results. The industry-level monthly sales revenue \( S(i, t) \) is seasonally adjusted and price adjusted throughout this paper as described in §3. The firm-level quarterly sales revenue is price adjusted throughout; it is not seasonally adjusted because of lack of benchmark seasonality factors. However, we use year-over-year changes in sales or dummy variables to control for seasonality as needed. All our results hold irrespective of whether the data are seasonally adjusted.

The sales change \( SC(i, t, T) \) represents the deviation of sales from past values over a given time window of length \( T \). As a default, we measure sales change over 12 months by setting \( T = 12 \), but in subsequent sections we vary \( T \) to assess how our results change with the length of the time window over which sales change is measured. Sales change may alternatively be measured as a rate of change, rather than the first difference. Although our results on amplification are consistent for either measure of sales change, a first-differenced model is mathematically more suitable for the analysis of aggregation of flows from downstream firms or industries. Therefore, we use a first-differenced sales change as our main measure and report the results using the rate of sales change for robustness.

**Definition 1.** Systematic risk in sales of firm or industry \( i \) for a given time window \( T \) is defined as \( \rho_S(i, T) = \text{Corr}(SC(i, t, T), r(t, T)) \).

The definition of systematic risk in sales thus has three components: sales change \( SC(i, t, T) \), the return on the market index \( r(t, T) \), and the time window \( T \). We estimate the systematic risk through the following regression equation for each firm or industry \( i \) and for each value of \( T \):

\[
SC(i, t, T) = a_i + b_i r(t, T) + \epsilon(i, t, T). \tag{1}
\]

Note that price and seasonal factors are adjusted in \( SC(i, t, T) \). Furthermore, systematic risk is insensitive to trends in sales and market-level data because \( SC(i, t, T) \) and \( r(t, T) \) are differences computed over a fixed time window. The error terms \( \epsilon(i, t, T) \) can be autocorrelated. Moreover, the Breusch and Pagan tests reveal a mild presence of heteroskedasticity for some industry segments. In the presence of heteroskedasticity and autocorrelation, ordinary least squares (OLS) estimation is consistent, but standard errors need to be adjusted to provide correct inferences. A standard approach is to use the Newey–West standard error estimator, which can account for an arbitrary heteroskedasticity and autocorrelation structure (Greene 2003, Chapter 10). Thus, we use OLS along with this approach and an autocorrelation structure with up to \( T \) lags.

Figure 1 illustrates the systematic risk at the sector level. Three stylized facts are evident from a visual inspection of the plots and are confirmed below by
formal computations: (1) The range of variation in the sales time series, i.e., the difference between the maximum and minimum values, widens as we move upstream. (2) The instantaneous volatility of the sales trajectory is higher for wholesalers than for retailers, and is the highest for manufacturers. (3) The sales trajectory follows the financial index progressively closer for the upstream sectors. It is apparent that the systematic risk is higher for industries that are upstream in the supply chain.

We present the estimates of $\rho_5$ for $T = 12$ months in the third column of Table 1. The aggregate estimate of $\rho_5$ is equal to 0.331 for retail trade, 0.415 for wholesale trade, and 0.502 for manufacturing. All three estimates are statistically significant at $p < 0.05$. Note the increase in estimates of $\rho_5$ from retail to wholesale trade to manufacturing sectors. The increase in the correlation coefficients is supported by the increase in covariance (see the fourth column). It is possible to build confidence intervals and test for statistical significance in differences between correlation coefficients using Fisher’s transformation of $\rho$ to $z$-scores (for details, see Chen and Popovich 2002). The difference between the correlation coefficient estimates for the retail and manufacturing sectors is significant at $p = 0.05$. The difference between the wholesale and retail sectors, and wholesale and manufacturing sectors has weak statistical significance ($p = 0.22$ and $p = 0.19$, respectively). Considering segment-by-segment estimates, we find that the model is significant at $p = 0.05$ for 2 of 6 retail, 9 of 18 wholesale, and 12 of 25 manufacturing nonoverlapping segments (Table 1).

5. Drivers of Systematic Risk

In this section, we posit three potential drivers for the increase in systematic risk upstream in the supply chain: within-industry production decisions, aggregation of flows across multiple customers, and aggregation of flows over time. The first driver is motivated by the literature on production smoothing and the bullwhip effect. The remaining two drivers relate to the structure of the supply chain. We intuitively explain the drivers in this section and present formal theoretical propositions for each driver in the appendix.

5.1. Systematic Risk in Production Decisions

Our first analysis is based on demand forecast updating or demand signal processing in supply chains. When a manager receives an order or a demand signal from her customer, she adjusts her own demand forecast based on that signal and thereby determines the production order to place with her upstream supplier. Such demand signal processing will lead to a transmission of systematic risk from downstream to upstream levels of the supply chain. Furthermore, it is possible that companies and industries respond more to systematic shocks and less to idiosyncratic in their demand signals. This increased response to systematic risk would lead to an amplification of the correlation coefficient of production with market returns. Thus, we test whether production data series of industries and firms have higher systematic risk than their sales data series.

Demand forecast updating has been studied in the literature as a driver of the bullwhip effect in which the variance of production of an industry is higher than the variance of sales. It has also been related to production smoothing, in which the variance of orders placed is lower than the variance of demand due to production capacity constraints or buffering effects of inventory. The phenomenon examined in our paper differs from variance amplification or attenuation. For instance, a firm that increases the covariance of its production series more than variance of its production series will exhibit both the bullwhip effect and amplification of systematic risk. On the other hand, a firm that increases the covariance of its production series less than variance of its production series will exhibit the bullwhip effect but not amplification of systematic risk. A similar reasoning also applies to production smoothing.

Following Cachon et al. (2007), we define the production of an industry $i$ in period $t$ as

$$P(i, t) = S(i, t) + B(i, t),$$

where $S(i, t)$ is the volume of price-adjusted shipments out of industry $i$ in month $t$, and $B(i, t) = I(i, t) - I(i, t - 1)$ is the inventory buildup in month $t$ ($I(i, t)$ is the end-of-month inventory of industry $i$ for month $t$). The production represents the inflow of material in industry $i$. Define the systematic risk in production as $\rho_5(i, T) = \text{Corr}(PC(i, t, T), r(t, T))$, where $PC(i, t, T) = P(i, t + T) - P(i, t)$ denotes the change in production over the time period $T$. Production will have greater systematic risk than sales if the uncertainty in inventory buildup is sufficiently correlated with $r(t, T)$. On the other hand, if inventory buildup is nonanticipatory of the market, or if noise drowns out the signal from the market, then systematic risk may decrease. We formalize this theoretical reasoning in Proposition 1 in the appendix.

5.2. Aggregation of Orders Over Customers

Another potential source of correlation amplification is the aggregation of orders coming from multiple customers. We propose that this effect occurs due to the network structure of supply chains in which suppliers serve varying number of customers.

An intuitive explanation of the aggregation effect is as follows. Each industry serves orders from many
other industries, and orders coming from each of those industries are potentially correlated with market returns in the sense described in §5.1. Order volume, a random variable, can be decomposed into two parts: one perfectly correlated with the market, and the other orthogonal to market returns. By pooling these orders, the standard deviation of the correlated components increases linearly, whereas the standard deviation of the orthogonal components increases as $\sqrt{N}$, where $N$ is the number of industries from which orders are pooled (assuming equal orders from all customers). This difference can lead to an increase in the correlation of the pooled orders with the market if the correlation of each order stream with the market is positive. Therefore, we expect sales at the upstream industry, which are obtained by pooling all downstream orders together, to have a higher correlation with market returns than sales at downstream industries.

To formalize this argument, consider a simplified two-level supply chain as in Figure 3(a). Let there be a single industry at the top level (industry 0) and $N > 1$ industries at the lower level, placing orders to industry 0. For each industry $i = 1 \ldots N$ at the lower level, let $S(i, t)$ denote the sales in period $t$, let $P(i, t)$ denote the production in period $t$, and let $PC(i, t, T)$ denote the production change over the time window $[t, t + T]$. Also let $\rho_P(i, T)$ denote the systematic risk in the production decisions of industry $i$ (see §5.1).

Consider industry 0 located upstream in the supply chain. Each of the downstream industries places orders to industry 0 to support its production. Assume that the amount of orders from industry $k$ in period $t$ is equal to $M_{k0} P(k, t)$, i.e., industry $k$ has to order $\$M_{k0}$ worth of materials from industry 0 to produce $\$1$ worth of output. Then the imputed sales at industry 0 in period $t$ are $S(0, t) = \sum_{k=1}^{N} M_{k0} P(k, t)$, and the imputed sales change is

$$SC(0, t, T) = \sum_{k=1}^{N} M_{k0} PC(k, t, T).$$

If $PC(k, t, T) = a_k^p + b_k^p r(t, T) + \varepsilon^p(k, t, T)$ according to §5.1, then

$$SC(0, t, T) = \sum_{k=1}^{N} a_k^p M_{k0} + r(t, T) \sum_{k=1}^{N} b_k^p M_{k0} + \sum_{k=1}^{N} M_{k0} \varepsilon^p(k, t, T).$$

The systematic risk of industry 0 computed via the imputed sales change is

$$\rho_{IO}(0, T) = \text{Corr}(SC(0, t, T), r(t, T)).$$

We now find that the necessary and sufficient condition for increase in systematic risk; i.e., $\rho_{IO}(0, T) > \rho_P(i, T)$, is given by

$$\frac{\left( \sum_{k=1}^{N} b_k^p M_{k0} \right)^2}{\left( b_i^p \right)^2} \geq \frac{\left( \sum_{k=1}^{N} b_k^p M_{k0} \right)^2 + \sum_{k=1}^{N} M_{k0} \left( \frac{\text{Var}(\varepsilon^p(k, t, T))}{\text{Var}(r(t, T))} \right)}{\left( b_i^p \right)^2 + \frac{\text{Var}(\varepsilon^p(i, t, T))}{\text{Var}(r(t, T))}}. \quad (3)$$

If the upstream industry serves identical customers, $\rho_{IO}(0, T)(N)$ increases in $N$, and $\lim_{N \to \infty} \rho_{IO}(0, T) = 1$. Therefore, it is possible that $\rho_{IO}(0, T) > \rho_P(k, T)$ for all $k$, or, equivalently, industry 0 has greater systematic

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**Figure 3** (Color online) (a) Supply Network with One Seller (Industry 0); $N$ Customers; (b) Imputed and Actual Sales for the Audio, Video, and Communication Equipment Manufacturing Segment (334×)

![Image](https://example.com/figure3.png)

**Note.** In panel (a), production orders from the customers are aggregated into imputed sales $S_k$. 

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risk than any of its customers. These results are formally stated in Proposition 2 and Corollary 1 in the appendix.

The above reasoning implies that industries serving multiple similarly sized small customers will have greater systematic risk than industries serving a smaller number of customers. Thus, we test the aggregation effect by hypothesizing that industries with higher customer-base dispersion, as measured by the inverse Herfindahl index, have greater systematic risk.

A few further observations are in order. First, $\rho_{IO}(0, T)$ and $\rho_S(0, T)$ are both measures of systematic risk in sales for industry 0, but are computed differently: $\rho_{IO}(0, T)$ is constructed entirely from data for downstream industries, without using the sales time series of industry 0, whereas $\rho_S(0, T)$ is computed from the sales data of industry 0. Generally, we find that the imputed sales are a good approximation to the actual sales. Figure 3(b) shows an example of the imputed and actual sales for the audio, video, and communication equipment manufacturing industry (M3 segments 34D, E, F). In §6, we further compute the values of $\rho_{IO}$ and $\rho_S$ for each industry and evaluate the difference between them. Second, it is possible that $\rho_{IO}(0, T) < \rho_P(i, T)$ if $b_i$’s have different signs. In principle, this suggests a way for a company to avoid a high correlation between the sales for their products and the market return: produce and sell a mix of products, the sales of which have different signs for $b_i$. In reality, the industries with negative $b_i$ are rare, as seen in Table 1. Such industries would play a role of insurance against drops in the financial market.

5.3. Aggregation of Orders Over Time

Thus far, we have considered the drivers of systematic risk for a given time window length $T$. In this section, we study how the systematic risk is affected by $T$. The sales change $SC(i, t, T)$ and the market return $r(t, T)$ in the definition of systematic risk can be measured over time windows of various lengths. Furthermore, their values over a longer time window can be decomposed into the sum of their values over shorter time windows. For instance, the sales change for $T = 12$ can be written as the sum of 12 monthly values of sales change. If the sales change over a period is correlated with not only the contemporaneous market return but also lagged market returns, then the systematic risk in sales may increase with the length of the time window.

To see this, Equation (1) can be rewritten as

$$SC(i, t, T) = S(i, t + T) - S(i, t + T - 1) + S(i, t + T - 1) - S(i, t + T - 2) + \ldots + S(i, t + 1) - S(i, t)$$

$$= \sum_{k=0}^{T-1} SC(i, t+k, 1)$$

$$= \sum_{k=0}^{T-1} [a_i + b_{i0}r(t+k, 1) + b_{i1}r(t+k-1, 1) + \ldots + b_{i(T-1)}r(t+k-T+1, 1) + \epsilon(i, t+k, 1)].$$

Here, we decompose $SC(i, t, T)$ into $T$ monthly changes and correlate each monthly change with that month’s contemporaneous market return as well as the market returns of the previous $T - 1$ months. By applying Definition 1 to this representation, we can now express the systematic risk in sales as a function of the coefficients $b_{i0}, \ldots, b_{i(T-1)}$. Thus, it is possible to examine the effect of time aggregation on the systematic risk by varying $T$.

If sales change is uncorrelated with lagged market returns, i.e., if $b_{i2} = \cdots = b_{i(T-3)} = 0$ in the above equation, then the systematic risk will be time invariant (Proposition 3 in the appendix). However, if this condition does not hold, then the systematic risk can increase with time aggregation. We test this hypothesis as the third potential driver of systematic risk.

In the appendix, we build theoretical model for time aggregation by showing sufficient conditions under which the systematic risk is time invariant or time varying. For instance, we formulate conditions for $|\rho(i, T)|$ to be an increasing function of $T$ in two special cases of the time-series structure—nondecaying lagged effects and geometrically decaying lagged effects (Propositions 4 and 5).

6. Evidence for the Drivers of Systematic Risk

In this section, we present statistical results based on the industry- and firm-level data for the three drivers of systematic risk postulated in §5. For each driver, we present the industry-level evidence first, followed by the firm-level evidence.

6.1. Systematic Risk in Production Decisions

Is the systematic risk in production time series larger than the systematic risk in sales time series? Table 1 presents the estimates of the systematic risk in production data, $\rho_P$, alongside the corresponding estimates of $\rho_S$ for $T = 12$ months. Note that $\rho_P < \rho_S$ based on the aggregate estimates for retail, wholesale, and manufacturing sectors, with estimates of 0.172, 0.290, and 0.443, respectively. Moreover, at a finer level of detail, using data for nonoverlapping segments, we find that $\rho_P < \rho_S$ for all 6 retail segments, 13 of 18 wholesale segments, and 20 of 25 manufacturing segments. These represent more than 50% of the segments in each case with $p < 0.05$. In fact, retailers, wholesalers, and manufacturers attenuate $\rho_P$.
compared to \( \rho_s \) by an average of 30%. Thus, the hypothesis that \( \rho_p > \rho_s \) is not supported by the data.

To see this inference differently, we plot \( \rho_p \) as a function of \( \rho_s \) for nonoverlapping segments in Figure 4(a). A linear fit of \( \rho_p \) on \( \rho_s \) shows that the slope coefficient (0.817) is significantly less than 1 (\( p < 0.05 \)). This estimate provides us two inferences. First, the systematic risk in production is significantly correlated with the systematic risk in sales. Second, systematic risk is attenuated in the production series compared to sales.

To address the question why \( \rho_p \) is not larger than \( \rho_s \), we examine the inventory buildup component of production decisions. We find that inventory buildup is weakly correlated with the market, with the correlation coefficient being approximately one-third of the systematic risk in production series. We find that inventory buildup is nonanticipatory with respect to the financial market conditions. In addition, \( \text{Var} \, PC(i, t, T) > \text{Var} \, SC(i, t, T) \) for most industries, indicating the presence of the bullwhip effect. This is consistent with the findings of Cachon et al. (2007), who document the bullwhip effect in the seasonally adjusted industry-level data. To summarize, \( \rho_p \) is less than \( \rho_s \) for most industries. Therefore, the industry-level data do not support the hypothesis that production decisions amplify systematic risk.

The firm-level data yield consistent results. We continue to observe an amplification of systematic risk from retailers to wholesalers and manufacturing. The mean values of \( \rho_s \) across firms in these industries are 0.23, 0.28, and 0.36, respectively, and the median values are 0.19, 0.35, and 0.41. We also compute the systematic risk in production decisions \( \rho_p(i, t, T) \) for each firm \( i \) and test whether it is amplified compared to the systematic risk in sales \( \rho_s(i, t, T) \). The average value of \( \rho_p \) across firms is 0.342 (standard deviation, 0.356), whereas the average \( \rho_s \) is 0.339 (standard deviation, 0.346). A test of the difference in means rejects the hypothesis that systematic risk in production is higher than that in sales. A pairwise comparison test also weakly rejects this hypothesis, as \( \rho_p < \rho_s \) for 104 of 238 firms. Figure 4(b) plots the values of \( \rho_p \) against \( \rho_s \) for the companies in our sample, in addition to the linear fit of \( \rho_p \) on \( \rho_s \). Overall, we observe that production decisions do not amplify systematic risk. This is consistent with the industry-level results.

### 6.2. Aggregation of Orders Over Customers

In this section, we examine whether customer dispersion is associated with higher systematic risk. We conduct this test in three ways. First, we estimate \( \rho_{IO} \) from downstream supply chain data and test whether the value of \( \rho_{IO} \) for an industry is a good approximation for the value of \( \rho_s \) for that industry. Second, we measure customer dispersion using the inverse Herfindahl index for each industry and estimate a statistical model to assess the effect of dispersion on systematic risk. Finally, we measure customer dispersion

![Figure 4](image-url)

(Color online) Systematic Risk in Production Decisions \( \rho_p \), Fitted Against Systematic Risk in Sales \( \rho_s \), for (a) Nonoverlapping Segments in Retail, Wholesale Trade, and Manufacturing and (b) Retail, Wholesale, and Manufacturing Public Companies, Constituents of the S&P 500 Index

<table>
<thead>
<tr>
<th>( \rho_p )</th>
<th>Coef.</th>
<th>Std. err.</th>
<th>LL</th>
<th>UL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_s )</td>
<td>0.817***</td>
<td>0.035</td>
<td>0.748</td>
<td>0.887</td>
</tr>
<tr>
<td>Intercept</td>
<td>−0.010</td>
<td>0.010</td>
<td>−0.030</td>
<td>0.011</td>
</tr>
</tbody>
</table>

\# obs. = 49, Prob. > F = 0, Adj. \( R^2 \) = 0.920.

Notes. The time window is \( T = 12 \) months. CI, confidence interval. LL and UL, lower and upper limit of the confidence interval, respectively.

\( * p < 0.1; ** p < 0.01 \).
at the firm level to test the same hypothesis in firm-level data.

The IO matrices can be used to reconstruct the flows of raw materials and products between industries and consumers. We derive the material requirements matrix \( M \), in which the \( ij \)th row shows the amount of inputs from all industries required to produce $1 output of industry \( i \). Using standard notation (see United Nations Statistical Office 1968 or Chen-trens 2007), let \( q \) denote a column vector of total commodity output, let \( g \) denote a column vector of total industry output, let \( U \) be the use matrix, and let \( V \) be the make matrix. The matrix \( U \) is a commodity by industry matrix that shows the amounts of commodities used by the industries as intermediates, and \( V \) is an industry by commodity matrix showing the amounts of commodities produced by each industry. Then, the material requirements matrix can be derived as

\[
M = (V \text{diag}^{-1}(q) U \text{diag}^{-1}(g)).
\]

The \( ij \) element of \( M \) shows the amount of input from industry \( j \) to industry \( i \) as

\[
M_{ij} = \frac{1}{g_j} \sum_k \frac{V_{ik}}{q_k} U_{kj},
\]

where \([k]\) represents the set of all commodities.

In the calculation of \( M \), we assume that each industry splits the flows for a commodity among all producer industries proportionally to their production shares of that commodity, and that the input structure for each industry is the same for its primary and secondary products. Both assumptions are standard in input-output analysis. We account for the final demand for products produced by each industry by introducing an extra row and an extra column into matrix \( M \). The last row represents the breakdown of $1 of final demand across all industries, and the last column is all zeros, representing the fact that no amount destined for final demand can be used as an intermediate.

Using the requirements matrix, if industry \( i \) produces \( x \) of output, then it should order \( xM_{ij} \) of raw materials from industry \( j \). Therefore, the total orders to industry \( j \) in period \( t \) are given by \( \sum_i M_{ij} P_{ij,t} \). We use \( M \) to compute \( \rho_{i0} \) as well as the inverse Herfindahl index for each industry.

### 6.2.1. Imputed Correlation-Based Evidence

Recall that \( \rho_{i0} \) is the systematic risk in imputed sales computed as follows:

\[
\rho_{i0} = \text{Corr}(\sum_i M_{ij} PC(i, t, T), r(t, T)).
\]

We present the estimates of \( \rho_{i0} \) and their relationship with \( \rho_s \) in the fifth and sixth columns of Table 2. The values of \( \rho_{i0} \) are estimated for the 41 nonoverlapping manufacturing segments, wholesale and retail trade, and the aggregated segment representing all other industries. Whereas \( \rho_p \) is smaller than \( \rho_s \), we find that \( \rho_{i0} \) is slightly larger than \( \rho_s \). To test the difference between \( \rho_{i0} \) and \( \rho_s \), we plot \( \rho_{i0} \) against \( \rho_s \) and fit a linear regression model (see Figure 5). The estimated model is statistically significant according to the F-test and has the adjusted \( R^2 \)-squared value of 0.39. At \( p = 0.05 \), the slope coefficient (0.856) is not significantly different from unity. The measure \( \rho_{i0} \) is constructed from downstream values of \( \rho_P \), yet it has a similar magnitude to \( \rho_s \). Thus, even though production does not amplify systematic risk in sales, the aggregation of production from downstream industries to upstream industries leads to the upstream industries having a higher systematic risk.

In summary, reconstructing and aggregating material flows in the supply chain network allows us to reconcile the attenuation of systematic risk in production decisions with the increase in systematic risk for industries upstream in the supply chain.

### 6.2.2. Industry-Level Panel Data Evidence

Recall from (3) that we would expect to see a higher \( \rho_s \) for an industry with a less concentrated customer base, or, equivalently, a higher dispersion of customer base. A commonly used measure of dispersion is the Herfindahl index; see, e.g., Libecap and Wiggins (1984). The inverse Herfindahl index for industry \( j \)'s customer base is given by

\[
H_j^{-1} = \left(\sum_i \left(\frac{M_{ij}g_i}{\sum_j M_{ij}g_i}\right)^2\right)^{-1}.
\]

Here, we choose the inverse Herfindahl index, rather than the direct Herfindahl index, because \( H^{-1} \) is more sensitive when customers are relatively small and the number of customers is large. In addition, if customers represent equal shares of demand, the inverse of Herfindahl index equals the number of customers. Consistent with the aggregation hypothesis, we expect to see a positive, statistically significant relationship between \( H^{-1} \) and the systematic risk.

We use the temporal and cross-sectional variation in \( H^{-1} \) to estimate the effect of customer concentration on systematic risk. The regression model is as follows:

\[
SC(i, t, T) = \alpha_1 r(t, T) + \alpha_2 H^{-1}(i, t) + \alpha_3 (r(t, T) \times H^{-1}(i, t)) + \alpha_4 SS(i, t) + \alpha_5 \text{INVT}(i, t) + \alpha_6 \text{GM}(i, t) + \alpha_7 \text{FD}(i, t) + \alpha_8 \text{ASV}(i, t) + \alpha_9 \tau(t) + \epsilon(i, t, T).
\]

We excluded the tobacco manufacturing segment from the analysis because of the heavy dependence between the demand for tobacco products and public health campaigns; see the note to Table 2.
This model captures the relationship between change in sales over a period of time, the contemporaneous return on the market portfolio, annual customer dispersion (measured by $H^{-1}$), and control variables. Customer dispersion is defined by (5) and computed annually using the corresponding material requirement matrix (4); $SS(i,t)$ is the standard deviation of sales, which we include to control for the magnitude of inventory and margin on sales (Kesavan et al. 2010); $FD(i,t)$ is the proportion of industry output used for final demand, included to control for the proximity of the industry to the final consumer; and $ASV(i,t)$ is the average sales volume in the past year, included to control for industry size. The model also includes an unobserved individual effect $u(i)$, and time trend $\tau(t)$. We estimate this model for $T = 12$ months, which yields a balanced panel with 43 industries and 15 annual observations per industry.

We hypothesize that industries with higher customer dispersion have higher systematic risk. This hypothesis implies that the coefficient $\alpha_3$ of the interaction effect $r \times H^{-1}$ between customer dispersion and market return should be positive and statistically significant. A positive coefficient would indicate that the sales of industries with a dispersed customer base are more exposed to the state of the economy, ceteris paribus. The summary statistics for the variables of model (6) are presented in Table 3 (top panel). We

**Table 3  Summary Statistics**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min</th>
<th>Max</th>
<th># obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Industry-level data, years 1993–2007</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Customer dispersion</td>
<td>Ratio</td>
<td>7.370</td>
<td>4.854</td>
<td>1.051</td>
<td>22.396</td>
</tr>
<tr>
<td>Sales</td>
<td>$\text{million/month}$</td>
<td>15,095.66</td>
<td>40,422.37</td>
<td>321.49</td>
<td>259,000</td>
</tr>
<tr>
<td>Sales change ($T = 12$ months)</td>
<td>$\text{million}$</td>
<td>520.949</td>
<td>2,073.8</td>
<td>-4,182.4</td>
<td>15,000</td>
</tr>
<tr>
<td>Rate of return ($T = 12$ months)</td>
<td>Ratio</td>
<td>0.108</td>
<td>0.146</td>
<td>-0.175</td>
<td>0.364</td>
</tr>
<tr>
<td>Sales std. dev.</td>
<td>$\text{million}$</td>
<td>446.116</td>
<td>834.944</td>
<td>5.836</td>
<td>5,661.224</td>
</tr>
<tr>
<td>Inventory</td>
<td>$\text{million}$</td>
<td>27,925.39</td>
<td>71,734.19</td>
<td>744.07</td>
<td>480,000</td>
</tr>
<tr>
<td>Gross margin</td>
<td>Ratio</td>
<td>0.25</td>
<td>0.47</td>
<td>0.17</td>
<td>0.45</td>
</tr>
<tr>
<td>Fraction of final demand in output</td>
<td>Ratio</td>
<td>0.317</td>
<td>0.350</td>
<td>-0.074</td>
<td>0.922</td>
</tr>
<tr>
<td>Avg. sales in past 12 months</td>
<td>$\text{million}$</td>
<td>14,718.21</td>
<td>39,067.1</td>
<td>323,551</td>
<td>253,750</td>
</tr>
<tr>
<td>Beta</td>
<td>Ratio</td>
<td>0.8678</td>
<td>0.3017</td>
<td>0.4403</td>
<td>1.7713</td>
</tr>
<tr>
<td><strong>Firm-level data, years 2009–2013</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of customers</td>
<td>Count</td>
<td>48.13</td>
<td>56.17</td>
<td>1</td>
<td>414</td>
</tr>
<tr>
<td>Customer dispersion</td>
<td>Ratio</td>
<td>5.23</td>
<td>5.08</td>
<td>1</td>
<td>27.47</td>
</tr>
<tr>
<td>Sales</td>
<td>$\text{million/Quarter}$</td>
<td>4,771.92</td>
<td>7,709.23</td>
<td>98.91</td>
<td>64,545.02</td>
</tr>
<tr>
<td>Sales change ($T = 4$ quarters)</td>
<td>$\text{million}$</td>
<td>84.53</td>
<td>1,467.04</td>
<td>-19,491.33</td>
<td>20,699.37</td>
</tr>
<tr>
<td>Rate of return ($T = 4$ quarters)</td>
<td>Ratio</td>
<td>0.052</td>
<td>0.280</td>
<td>-0.0584</td>
<td>0.459</td>
</tr>
<tr>
<td>Sales std. dev.</td>
<td>$\text{million}$</td>
<td>368.56</td>
<td>766.34</td>
<td>3.17</td>
<td>10,234.43</td>
</tr>
<tr>
<td>Inventory</td>
<td>$\text{million}$</td>
<td>1,935.26</td>
<td>2,873.70</td>
<td>0</td>
<td>38,723.16</td>
</tr>
<tr>
<td>Gross margin</td>
<td>Ratio</td>
<td>0.41</td>
<td>0.23</td>
<td>0</td>
<td>0.96</td>
</tr>
<tr>
<td>Num. of firms in the same NAICS3</td>
<td>Count</td>
<td>23.8</td>
<td>17.85</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>Avg. sales in past 4 quarters</td>
<td>$\text{million}$</td>
<td>4,882.94</td>
<td>7,806.59</td>
<td>187.89</td>
<td>70,379.75</td>
</tr>
<tr>
<td>Beta</td>
<td>Ratio</td>
<td>1.0584</td>
<td>0.4402</td>
<td>0.2718</td>
<td>2.2565</td>
</tr>
</tbody>
</table>
estimate two versions of the model: the base version without the interaction effect $r \times H^{-1}$, and the full version including it.

In the base case, the market return is the only statistically significant predictor positively associated with the sales change. In the full model, the coefficient on the interaction effect between market return and customer-base dispersion is positive and highly significant, whereas the coefficient on the market return is not significant (Figure 6). This suggests that the effect of market return on sales change is mediated by the customer dispersion: a more diversified customer base is associated with a greater effect of market conditions on sales.

### 6.2.3. Firm-Level Evidence

We extend the analysis of customer dispersion using firm-level data. The data include sales revenue breakdown by major customers, which allows us to compute the customer-base dispersion. Our model specification is similar to (6):

$$SC(i, t, T) = \alpha_1 r(t, T) + \alpha_2 H^{-1}(i)$$

$$+ \alpha_3 (r(t, T) \times H^{-1}(i)) + \alpha_4 SS(i, t)$$

$$+ \alpha_5 INV(i, t) + \alpha_6 NCUST(i) + \alpha_7 GM(i, t)$$

$$+ \alpha_8 NFIRM(i, t) + \alpha_9 ASV(i, t) + \alpha_{10} \tau(t)$$

$$+ \alpha_{11} NAICS2(i) + \epsilon(i, t). \quad (7)$$

Here $ASV(i, t)$ is the average sales volume in the past four quarters (size proxy), $NFIRM(i, t)$ is the number of firms in the same NAICS3 segment (proxy for competition), $NCUST(i)$ is the number of customers of firm $i$, and $H^{-1}$ and $NCUST$ are assumed to be time invariant during years 2009–2013. NAICS2 is a two-digit NAICS segment effect. The rest of the variables are defined as in (6). Sales and inventory data are adjusted for price inflation. We report the estimation results of model (7) in Figure 6.

We estimate two versions of the model: the base version without the interaction effect $r \times H^{-1}$ and the full version including it. In the base case, the market return and customer dispersion are positively associated with the sales change; however, the latter is only marginally significant. In the full model with the interaction between customer dispersion and market return, the coefficient on the interaction effect is positive and highly significant, whereas the coefficient on the market return is weakly significant. Comparing the magnitude of the effect of $H^{-1}$ at the firm and industry levels, we find that it is similar. The number of customers is positively associated with the systematic risk in both models. Collectively, this establishes the relationship between the systematic risk and customer dispersion at the firm level and supports the conclusion reached at the industry level: a more diversified customer base is associated with a greater effect of systematic risk.

Thus, we find evidence supporting that customer-base dispersion is a driver of increase in systematic risk in supply chains. We have shown this inference in three ways. Using $\rho_{10\tau}$, we showed that although industries attenuate systematic risk in their production series, customer-base dispersion leads to sufficient amplification to offset the effect of attenuation of systematic risk in production series. Second, using...
a panel data model, we showed that industries with a more dispersed customer base exhibit higher systematic risk. Finally, we showed the same result on customer dispersion using firm-level data.

### 6.3. Aggregation of Orders Over Time

To test the third driver of systematic risk, aggregation of demand over time (5.3), we estimate $\rho_s$ over time windows of varying length $T$. The choice of time window represents the duration of time windows over which lagged market return can be correlated with current period sales change.

Figure 7 plots the estimates of correlation coefficients $\rho_s$ for retailers, wholesalers, and manufacturers, for $3 \leq T \leq 36$. The difference between correlation coefficients for retailers and wholesalers is significant at $p = 0.05$ for $T \geq 14$, and the difference for retailers and manufacturers is significant for $T \geq 10$. The significance level for the difference between wholesalers and manufacturers is about $p = 0.25$ for $9 \leq T \leq 18$ and increases to $p = 0.05$ for $T \geq 24$. Note that $\rho_s$ reaches the highest level for manufacturers at $\approx 0.85$, followed by wholesalers at $\approx 0.65$, and retailers at $\approx 0.35$. In the case of retailers, $\rho_s$ exhibits a downward trend and declines from $\approx 0.35$ to $\approx 0.10$ for $30 \leq T \leq 36$, with the negative trend continuing for larger values of $T$.

Regressing $|\rho_s(i, T)|$ on $T$ for the 49 nonoverlapping industry segments, with industry-segment fixed effects, and $T$ varying from 3 to 36 months, we obtain that systematic risk increases by 0.012 for each month increase in $T$ on average. The increasing trend suggests the presence of lag effects of returns on sales change. In fact, fitting the model with lagged returns (§5.3) results in insignificant effects of lags for retailers, weakly significant effects for wholesalers, and highly significant effects for manufacturers. This suggests that the financial market effect dissipates slower in time for manufacturers than for wholesalers and retailers.

At the firm level, we compute $\rho_s(i, T)$, for $T$ varying from 1 to 12 quarters, and each company, indexed by $i$. Then we test whether $|\rho_s(i, T)|$ increases with $T$, regressing $|\rho_s(i, T)|$ on $T$ with firm-level fixed effects. Figure 7 reports the results. An increase in $T$ by one quarter (three months) is associated with the increase in $|\rho_s|$ by 0.028. The positive dependence on $T$ supports the results of the industry-level analysis.

Overall, we observe that aggregation over time increases the systematic risk. The effect is more pronounced for manufacturers and wholesalers. Thus, decisions under uncertainty with longer time horizons will entail higher systematic risk.

### 6.4. Robustness Checks

We describe two further robustness checks of our results in this section. First, we extend the specifications (6) and (7) by including additional interaction effects. Second, we test alternative measures of sales change.

We extend the specifications (6) and (7) by including interaction effects between $r$ and all other continuous variables. At the industry level, our main variable of interest, the interaction $r \times H^{-1}$, is positive and significant, with $p = 0.03$. Other interactions are also statistically significant at $p < 0.1$ or better. In particular, interactions with sales standard deviation, size, and gross margin are positive, suggesting that those factors amplify systematic risk. Interactions with inventory and proximity to final demand are negative, suggesting that those factors dampen systematic risk. At the firm level, the interaction $r \times H^{-1}$ is positive and

![Figure 7](Image)

**Figure 7** (Color online) Systematic Risk as a Function of $T$ with 90% Confidence Intervals, for Retail, Wholesale Trade, and Manufacturing Segments

<table>
<thead>
<tr>
<th>Industry-level panel with 49 industries</th>
<th>Coef.</th>
<th>Std. err.</th>
<th>95% CI</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\rho_s</td>
<td>$</td>
<td>$T$ (months)</td>
<td>0.012***</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.09***</td>
<td>0.005</td>
<td>0.082</td>
<td>0.100</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.256</td>
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<th>Firm-level panel with 239 firms</th>
<th>Coef.</th>
<th>Std. err.</th>
<th>95% CI</th>
<th>95% CI</th>
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<td>\rho_s</td>
<td>$</td>
<td>$T$ (quarters)</td>
<td>0.028***</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.267***</td>
<td>0.006</td>
<td>0.254</td>
<td>0.279</td>
</tr>
<tr>
<td>$R^2$</td>
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</tr>
</tbody>
</table>

**Notes.** The figure shows industry- and firm-level regression analysis of the systematic risk and $T$. CI, confidence interval. LL and UL, lower and upper limit of the confidence interval, respectively.

*** $p < 0.01$.  

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significant at \( p = 0.016 \); all other interactions are not significant, with \( p > 0.3 \), except for \( r \times SS \), which has \( p = 0.063 \). Collectively, this evidence supports the role of \( H^{-1} \) in the amplification of systematic risk beyond size, margin, and other effects.

In addition, we repeat all of the empirical analysis with sales change defined as \( \ln S(i, t + T) - \ln S(i, t) \). This measure is insensitive to the size of \( S \). The estimates of systematic risk under this alternative measure are consistent with those reported previously: at the industry level they are 0.41, 0.45, and 0.49, and the means of the estimates at the firm level are 0.22, 0.29, and 0.36, for retail, wholesale trade, and manufacturing, respectively (\( T = 12 \) months). We continue to observe the attenuation of the systematic risk in production decisions by 22% at the industry level, and to a lesser degree, by 8%, at the firm level, with both models statistically significant at \( p < 0.01 \). We also continue to observe the amplification of systematic risk due to aggregation over customer base. At the industry level, \( \rho_{10} \) remains an unbiased predictor of \( \rho_{S} \). At both industry and firm levels, customer-base dispersion continues to mediate the effect of the market return on the sales change, with a positive and statistically significant interaction \( r \times H^{-1} \). Among other interactions, only interactions with the standard deviation of sales and gross margin remain positive and statistically significant at the industry level. At the firm level, interactions with number of customers and the standard deviation of sales are associated with increased systematic risk. Finally, we also continue to observe the amplification of systematic risk with \( T \), with \( \rho_{S} \) increasing by 0.015 for a one-month increase in \( T \) on average for the industry-level data. At the firm level, \( \rho_{S} \) increases by 0.13 for every one-quarter increase in \( T \). Finally, the results presented in §7.1 are robust to the alternative definition of systematic risk.

7. Managerial Implications

In this section, we test whether the systematic risk in sales is priced by the financial market, i.e., whether the increased correlation between sales change and return on the market portfolio translates into increased beta for the associated industry portfolio or stock. In addition, we discuss how systematic risk and lead time affect the value of future payoffs. We show how to refine a risk-adjusted valuation of such payoffs based on our empirical estimates. Finally, since sourcing decisions change the customer base of a supplier, we discuss the implications of sourcing decisions for supplier’s risk.

7.1. Cost of Capital

Consider an example firm from our study. The capital asset pricing model connects the risk premium of the firm’s equity and its covariability with the market.

Under the capital asset pricing model,

\[
\mathbb{E}(r_i) = r_f + (\mathbb{E}(r) - r_f)\beta_i, \tag{8}
\]

where \( r_i \) is the return on portfolio of industry or firm \( i \), \( r \) is the return on the market portfolio, \( r_f \) is the risk-free rate, and \( \beta_i = \text{Cov}(r, r_i) / \text{Var}(r) \) is the beta of the portfolio \( i \). Equation (8) can be rewritten as

\[
\mathbb{E}(r_i) = r_f + \frac{\mathbb{E}(r) - r_f}{\sigma_r} \text{Corr}(r_i, r)\sigma_i, \tag{9}
\]

where \( \sigma_i \) and \( \sigma_r \) are standard deviations of \( r_i \) and \( r \), respectively.

Equations (9) and (1) are similar. Indeed, (1) can be rewritten as

\[
\mathbb{E}(\text{SC}(i, t, T)) = a_i + \frac{\mathbb{E}(r(t, T))}{\sigma_r} - \rho_S(i, t) \text{Var}^{1/2}(\text{SC}(i, t, T)).
\]

The former characterizes the effect of the market on expected return, and the latter the effect on the market on expected sales change. It is likely that sales change (expressed in relative terms) and returns are also correlated, because earnings are generated from the sales revenue of a firm. In fact, if \( r_i \) over \([t, t + T]\) is perfectly correlated with \( \text{SC}(i, t, T)/\mathbb{E}(\text{SC}(i, t)) \), then \( \beta_i \sim \rho_S(i, t) \times \sigma_S(i) \), where \( \sigma_S(i) = \text{Var}^{1/2}(\text{SC}(i, t, T))/\mathbb{E}(\text{SC}(i, t)) \). This motivates us to test statistical relationship between \( \rho_S, \sigma_S \), and beta.

We proceed as follows. At the industry level, we construct portfolios that include every public company in a given industry segment on any given day from January 1992 to December 2007; \( \rho_S \) and \( \sigma_S \) are computed for the same period of time. The composition of portfolio on a given day is determined by public companies that are in business on that day, and thus changes over time. The share of each company in the portfolio is proportional to its market capitalization on the previous trading day. We use value-weighted portfolios because the estimates of \( \rho_S \) are based on aggregate sales data obtained by adding sales of all companies in an industry. Then we compute daily returns on the portfolios, and estimate betas using (8). We use daily returns on the VWMI (see Cochrane 2001, §12.1) and assume a constant risk-free rate. Estimates of beta for industry portfolios are presented in Table 2. At the firm level, we use the average value of annual company betas for years 2009–2013 obtained from the Center for Research in Security Prices and compute \( \rho_S \) and \( \sigma_S \) for the same period. In both industry- and firm-level analyses, we use \( T = 12 \) months.

We test statistical relationship of beta with \( \rho_S \) and \( \sigma_S \) using the following model:

\[
\beta_i = \alpha + \beta_1 \rho_S(i) + \beta_2 \sigma_S(i) + \beta_3 (\rho_S(i) \times \sigma_S(i)) + \epsilon_i, \tag{10}
\]
We also estimate the basic relationship between $\beta$ and $\rho_S$, where $b_2 = b_3 = 0$. The results are presented in Table 4.

At the industry level, both the base and full model are highly statistically significant according to the $F$-test. The industry-level estimate of coefficient on $\rho_S$ in the base model is 0.65 ($p < 0.05$). The model explains approximately 10% of variance in betas. The full model (10) explains more than 40% of variance in betas. The coefficient on $\rho_S \times \sigma_S$ is positive and highly significant, suggesting that the beta is proportional to the systematic risk in sales and sales volatility. The firm-level results are similar to the industry-level results: in the base model, the estimate of the coefficient on $\rho_S$ is 0.42 ($p < 0.05$) and the coefficient on $\rho_S \times \sigma_S$ is positive and significant.

To summarize, the results of both models support the hypothesis that financial markets price the systematic risk in sales revenue. If the equity risk premium is 6% (Damodaran 2012), the estimates of the relationship between beta and $\rho_S$ suggest that for every increase in $\rho_S$ by 0.1, the cost of equity increases by 0.25%–0.39% per annum.

### 7.2. Risk-Adjusted Valuation

We first review the standard project valuation framework (cf. Copeland and Antikarov 2001, Chapter 3) and then illustrate it using our empirical estimates. Without loss of generality, assume that the project investment occurs at time $t = 0$, and the investment generates a random payoff $S$ at time $T$. According to the capital asset pricing model, the risk-adjusted value of the cash flow at time 0 is given by

$$V_0(T) = \frac{\mathbb{E}(S) - \Omega \text{Cov}(S, r(T))}{1 + r_f},$$

where $\Omega = \mathbb{E}(r) - r_f / \text{Var}(r)$ is the market price of risk, $r$ is the return on the market portfolio invested at time 0 at time $T$, and $r_f$ is the risk-free rate. Note that $\text{Cov}(S, r(T)) = \rho_S(T) \sqrt{T} \text{Var}^{1/2} S \text{Var}^{1/2} r$ is a function of systematic risk and $T$.

Figure 8 presents the risk-adjusted value $V_0(T)$ for an example investment with the expected payoff of 100, with a standard deviation of 10. The risk-adjusted value depends on the systematic risk of the industry where investment is made. We use the industry-level estimates to compute $V_0(T)$ and use the

![Figure 8](image-url)
7.3. Supplier Risk
A decision to source from a supplier changes the supplier’s customer base. Our analysis from §6.2.2 shows that the systematic risk of the supplier will change as a result of this decision. We present this implication in this section.

The parsimonious representation of (7) can be written as

\[
SC(i, t, T) = \alpha_1 r(t, T) + \alpha_2 r(t, T) \times H^{-1}(i, t) + u(i) + \epsilon(i, t),
\]

which results in the following expression for the systematic risk:

\[
\hat{\rho}_S(H^{-1}) = \left(1 + \frac{\sigma_u^2 + \sigma_e^2}{(\alpha_1 + \alpha_3 H^{-1})^2 \text{Var}r}\right)^{-1/2}. \tag{11}
\]

Function (11) is plotted in Figure 6 using the estimates on the left panel of the figure. Function (11) is increasing and concave. In the firm-level range of data \(1 \leq H^{-1} \leq 27\), the relationship is close to linear; i.e., the systematic proportionally increases with \(H^{-1}\).

At the mean value of customer dispersion, the estimate of systematic risk is \(\hat{\rho}_S(5.24) = 0.19\). The range of systematic risk associated with changing \(H^{-1}\) by up to one standard deviation (\(\text{Std}(H^{-1}) = 5.09\)) is from 0.10 to 0.28.

To see the implications of this increase in \(H^{-1}\), suppose that a supplier ships equal amounts to five customers, and a new customer joins the customer base placing the same order size as the other customers. Using a firm-level estimate, we find that the addition of the new customer will increase the systematic risk of the supplier by 11% (from 0.18 to 0.20), ceteris paribus. Correspondingly, as per §7.1, the supplier’s cost of capital will increase by approximately 1%. (In relative terms, the associated beta increases from 0.99 to 1, using the firm-level base model; see Table 4.) Per §7.2, risk-adjusted values of investments’ into the supplier company will also decrease.

When making a sourcing decision, buyers need to be aware of its impact on their suppliers’ systematic risk. In general, a decision to source from a supplier can lead to either an increase or decrease in the supplier’s systematic risk. If the buyer is large (compared to existing customers), then the supplier’s customer base can become more concentrated; thus, the systematic risk will decrease. If the buyer is similar in size to existing customers of that supplier, then the supplier’s systematic risk will increase. Thus, customer-base audit and evaluation of systematic risk of suppliers should be a part of due diligence for sourcing decisions.

8. Discussion
The contribution of this paper is to show that systematic risk is an important phenomenon for industries and firms and that it is affected by the supply chain structure. We show that the customer base concentration and length of time horizon are determinants of the systematic risk. In particular, an increased time horizon and aggregation of orders over a large number of customers are both associated with a greater systematic risk. The greater systematic risk is in turn associated with the increased cost of capital and reduced risk-adjusted value of a business.

Throughout this paper, we measure systematic risk using material flow data. The recent literature has distinguished between the estimation of the bullwhip effect using material flow versus information flow data. Chen et al. (2014) show for a stylized product-level model of an order-up-to policy with backlogging of excess demand that the bullwhip effect measured from material flow data is larger than the bullwhip effect measured from information flow data. The choice between information flow versus material flow data could affect the estimates of systematic risk as well. Thus, we simulate the model of Chen et al. (2014) modified to allow demand to be correlated with market return. We find that the systematic risk measured using material flows is less than the systematic risk measured using informational flows, and is a close approximation when fill rates are high. The usage of information flows for the estimation of systematic risk in sales and production may be examined in depth in future research.

Our result is relevant for firms in the context of the financial crisis in 2008 and 2009 and the ensuing economic recession. Since demand for all products dips during recession, flexibility strategies that seek to hedge against idiosyncratic noise are less valuable. For example, Toyota hedges variability in global demand by meeting excess demand from a flexible plant in Japan (see Iyer et al. 2009). The plant was forced to remain idle when the global economy slumped. Operational flexibility is advantageous in such situations only if it provides a hedge against
the systematic risk component of demand uncertainty. For example, postponement flexibility, i.e., lead time reduction, can be more beneficial than flexible production capacity to mitigate the increase in systematic risk. Moreover, firms that produce products with different degrees of systematic risk will benefit more from operational flexibility.

Our findings also suggest that firms should be careful when selecting their suppliers. In particular, due diligence should be exercised not only with respect to the supplier’s business, but also its customers. A firm’s business may be subject to supplier risk not because of the supplier firm itself, but because of the supplier’s other customers. In that respect, sourcing from a dedicated supplier can be justified, despite the increased cost. For suppliers, the presence of the systematic risk changes their view on the traditional operational hedging techniques. Geographical demand pooling strategies would be less effective for managing the systematic risk. Demand pooling provides a hedge against idiosyncratic noise because noise in the demand of products or across geographical locations will cancel out on average. But such strategies are not useful against systematic risk, which will not cancel out.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2015.2187.

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Appendix. Formal Statements and Proofs

Proof of Proposition 1. Let $BC(i, t, T) \overset{\Delta}{=} B(i, t + T) - B(i, t) = a_i^b + b_i^B r(t, T) + e^B(i, t, T)$ for some fixed $T$; $e^B(i, t, T)$ is drawn from the same distribution as $\kappa e(i, t, T)$ for some $\kappa \geq 0$, $e(i, t, T)$ is as (1), and $\text{Corr}(e^B(i, t, T), e(i, t, T)) \geq 0$. If $b_i^B > \kappa b_i$, then $\rho_B(i, T) > \rho_S(i, T)$.

Proof of Proposition 2. Let $PC(i, t, T) = P(i, t + T) - P(i, t) = SC(i, t, T) + BC(i, t, T)$

$= a_i^b + b_i^B r(t, T) + e(i, t, T) + e^B(i, t, T)$,

$\rho_S(i, T) = \left(1 + \frac{\text{Var}(e(i, t, T))}{b_i^B \text{Var}(r(t, T))}\right)^{-1/2}$, and

$\rho_B(i, T) = \left(1 + \frac{\text{Var}(e(i, t, T) + e^B(i, t, T))}{(b_i^B + b_i^B)^2 \text{Var}(r(t, T))}\right)^{-1/2}$.

$\rho_B \geq \rho_S$ if and only if $\text{Var}(e(i, t, T)) \geq \text{Var}(e(i, t, T) + e^B(i, t, T))$ because $e^B(i, t, T)$ and $\kappa e(i, t, T)$ are identically distributed and nonnegatively correlated.

Hence, $\rho_B \geq \rho_S$ if $b_i^B / b_i \geq \kappa$. Intuitively, if $b_i^B$ is large, the production is anticipative of the market, or if $\kappa$ is small, the noise term does not drown out the signal from the market.

Proof of Proposition 2. Let $PC(i, t, T) = P(i, t + T) - P(i, t) = a_i + b_i r(t, T) + e(i, t, T)$. The systematic risk of industry $0$ is amplified with respect to industry $i$ production, i.e., $\rho_{i0}(0, T) \geq \rho_{i}(i, T)$, if and only if

$$\frac{(\sum_{k=1}^{N} b_k M_{k0})^2}{b_i^B} \geq \frac{(\sum_{k=1}^{N} b_k M_{k0})^2 + \sum_{k=1}^{N}(M_{k0})(\sigma_i / \sigma_i)^2}{b_i^B + \sigma_i^2 / \sigma_i^2}.$$
Corollary 1. If industry 0 serves N identical customers, i.e., $b_k = b$, $\alpha_k = \sigma$, and $M_k = M_0$ for all $k = 1, \ldots, N$, then $\rho_{00}(0, T)(N)$ increases in $N$ and $\lim_{N \to \infty} \rho_{00}(0, T)(N) = 1$.

Proof of Corollary 1. Note that (3) can be rewritten as $1 \geq (1 + (1/N)\sigma^2/(\beta^2 \sigma^2))(1 + \sigma^2/(\beta^2 \sigma^2))$, which holds trivially. Indeed, in this case,

$$p_{00}(0, T)(N) = \frac{b \sigma^2}{\sqrt{\beta^2 \sigma^2 + \sigma^2/N}} = p_0(k, T) \quad \text{for all } k.$$

Note that $\rho_{00}(0, T)(N)$ is increasing in $N$ and $\lim_{N \to \infty} \rho_{00}(0, T) = 1$. Therefore, it is possible that $\rho_{00}(0, T) > p_0(k, T)$ for all $k$, or equivalently, industry 0 has greater systematic risk than any of its customers. □

Proposition 3. If $SC(t+k, 1) = a + b \text{Var}(t+k, 1) + \epsilon(t+k, 1)$ and $\epsilon(t+k, 1)$ is independent from $r(t+k-1, 1), r(t+k-2, 1), \ldots$ for all $k = 0, \ldots, T-1$, then $\rho_S(T) = p_0(1)$.

Proof of Proposition 3. Note that $\rho_S(1) = \text{Corr}(SC(t, 1), r(t, 1)) = \text{Var}(\text{Corr}(SC(t, 1), r(t, 1)))$,

$$\rho_S(T) = \text{Corr}(SC(t, 1), r(t, T)) = \text{Corr}(T \sum_{k=0}^{T-1} [a + b \text{Var}(t+k, 1) + \epsilon(t+k, 1)], T \sum_{k=0}^{T-1} r(t+k, 1)) = \frac{T \text{Var}(\text{Corr}(SC(t, 1), r(t, T)))}{\sqrt{T} \text{Var} \text{Corr}(SC(t, 1), r(t, T))} = \rho_S(1).$$

Proposition 4. Let $b_0 = b_1 = \cdots = b_T = b$; $\epsilon(t+k, 1)$ is independent and identically distributed (i.i.d.), and $r(t+k, 1)$ are i.i.d. for $k = 0, \ldots, T$. If

$$\frac{\gamma^2}{\beta^2} = \frac{8T^3 + 18T^2 + 10T - 3}{9 + 6T},$$

where $\gamma^2 = \text{Var}(\epsilon(1, 1)/\text{Var}(r(1, 1))$, then $|\rho_T(T+1)| > |\rho_T(T)|$. Moreover, $\lim_{T \to \infty} \rho_T(T) = \text{sign}(b) \sqrt{3}/2 \sqrt{2}$.

Proof of Proposition 4. In general,

$$\rho_S(T) = \left(\frac{T \text{Var}(\text{Corr}(SC(t, 1), r(t, T)))}{\sqrt{T} \text{Var} \text{Corr}(SC(t, 1), r(t, T))}\right) \cdot \sqrt{\left(\frac{T \text{Var}(\text{Corr}(SC(t, 1), r(t, T)))}{\sqrt{T} \text{Var} \text{Corr}(SC(t, 1), r(t, T))}\right)^2 + \text{Var}(\text{Corr}(SC(t, 1), r(t, T)))^2}.$$  

When $b_0 = b_1 = \cdots = b_T = b$,

$$\rho_S(T) = \frac{(T+1)b/2}{\sqrt{(T-1)(2T-1)\beta^2/3 + T \beta^2 + \gamma^2}},$$

$$\rho_S(T+1) = \frac{(T+2)b/2}{\sqrt{(T+1)b^2/3 + (T+1)\beta^2 + \gamma^2}}.$$

The limiting result now follows. After simplifications, $|\rho_T(T+1)| > |\rho_T(T)|$ if and only if

$$(2T+3) \left(\frac{b^2}{3} (T-1)(2T-1) + T \beta^2 + \gamma^2\right) > 4(T+2)T \beta^2,$$

or

$$\frac{\gamma^2}{\beta^2} > \frac{8T^3 + 18T^2 + 10T - 3}{9 + 6T}.$$

□

Proposition 5. Let $b_k = \alpha_k b$, $0 \leq \alpha < 1$; $\epsilon(t+k, 1)$ are i.i.d., and $r(t+k, 1)$ are i.i.d for $k = 0, \ldots, T$. Then

$$|\rho_T(T+1)| > |\rho_T(T)|$$

as $T \to \infty$. Moreover, $\lim_{T \to \infty} \rho_T(T) = \text{sign}(b)(1 + \sqrt{1 - \alpha^2})^{2}\beta^2 - 1/2$.

Proof of Proposition 5. Let $\gamma^2 = \text{Var}(\epsilon(1, 1)/\text{Var}(r(1, 1))$. If $b_k = \alpha_k b$, using the partial sum formula $\sum_k \alpha^k/(1-\alpha)$,

$$\rho_T(T) = \text{sign}(b) \left(\frac{T+1}{1-\alpha} - \frac{1}{(1-\alpha)^2}\right) \cdot \sqrt{T} \left(\frac{1}{1-\alpha^2} - \frac{1}{(1-\alpha)^2} [T(1 - \alpha^2 + 2\alpha^2 \epsilon^2 - \alpha^2 T + 2) - 1 + \alpha^2 - 2\alpha^2 + 2\alpha^2 T + 2] \frac{T}{\beta^2}\right)^{-1/2}.$$

Consider the absolute value $|\rho_T(T)|$ as $T \to \infty$.

$$|\rho_T(T)| = \text{sign}(b) \left(\frac{T+1}{1-\alpha} - \frac{1}{(1-\alpha)^2}\right) \cdot \sqrt{T} \left(\frac{1}{1-\alpha^2} - \frac{1}{(1-\alpha)^2} [T(1 - \alpha^2) - 1 + \alpha^2 - 2\alpha^2 + 2\alpha^2 T + 2] \frac{T}{\beta^2}\right)^{-1/2},$$

that is, $|\rho(T)|$ approaches a rational function as $T \to \infty$. If

$$T > (2\alpha^2)/(1-\alpha^2)^2 + \frac{2\alpha^2}{(1-\alpha^2)^2} + \frac{2\alpha^2}{(1-\alpha^2)^2},$$

the function is increasing in $T$ with the asymptote $(1 + \gamma^2)/(\beta^2) - 1/2$. □

References


